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Extension of worldline computational algorithms for QCD to open fermionic contours

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Abstract: The worldline casting of a gauge field system with spin-1/2 matter fields has provided a, particle-based, first quantization formalism in the framework of which the Bern-Kosower algorithms for efficient computations in QCD acquire a simple interpretation. This paper extends the scope of applicability of the worldline scheme so as to include open fermionic paths. Specific algorithms are established which address themselves to the fermionic propagator and which are directly applicable to any other process involving external fermionic states. It is also demonstrated that in this framework the sole agent of dynamics operating in the system is the Wilson line (loop) operator, which makes a natural entrance in the worldline action; everything else is associated with geometrical properties of particle propagation, of which the most important component is Polyakov's spin factor.

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1. Introduction

Consider a gauge theory with spin-1/2 matter fields, in the defining representation of the symmetry group, described by a standard (renormalizable) lagrangean density. A worldline transcription of such a system is based, after a Wick rotation to euclidean space-time, on a recasting of the form $\int \mathcal{D}\overline{\psi}(x)\,\mathcal{D}\psi(x)\,e^{S[\overline{\psi}(x),\psi(x),A_\mu(x)]}\ldots \longrightarrow \int \mathcal{D}x(\tau)\,\mathcal{D}p(\tau)\,e^{S[x(\tau),p(\tau),A_\mu(x(\tau))]}\ldots$ according to which the matter fields register as point-like objects whose propagation (in phase-space) is furnished by a path integral. To the extent that the functional integration over the fermionic fields is gaussian (in the Grassmann sense), nothing is lost, in principle, from the content of the original system through the above transcription. Pioneering work that has dealt systematically with the subject is found in refs. [1]–[3].

More recent advancements in worldline methodology, especially in connection to gauge theories, have been made, independently, by Fried [4]–[6], Strassler [7] and our group [8]–[12]. Respective methodological tools adopted by the above authors are: (a) Schwinger's functional calculus formalism [13], (b) the string-inspired computational algorithms of Bern and Kosower [14] and (c) Polyakov's description of the propagation of particle-like excitations in (euclidean) space-times [15]. Concerning specific objectives and/or applications aspired to by these attempts, the following rough picture emerges. Fried and collaborators [4]–[6] as well as the present authors and collaborators [10]–[12] have so far focused their efforts on eikonal-type descriptions in gauge field theories (with QCD as the ultimate target), along with associated issues such as scattering amplitudes at asymptotically large collision energies, factorization of soft physics, heavy quark effective theories, etc. In Strassler's case, on the other hand, emphasis from the beginning was placed on the development of new, more efficient rules for performing loop calculations in pQCD as per the realizations of ref. [14].

A key aspect of the Polyakov-inspired approach, that we have been advocating, is its ability to accentuate geometrical features of particle-like propagation in space-time. To illustrate this occurrence consider, first, the direct performance of the path integration over $\mathcal{D}p(\tau)$.[1] Adhering to the use of the first order form for the Dirac operator we were able [9] to reproduce Polyakov's *spin factor* which characterizes the closed path propagation of a spin-1/2 particle-like entity in euclidean space-time. This quantity has a purely geometrical content (it is, in some sense, associated with torsion) and accounts in a self-consistent way, for the re-entry of a closed (euclidean) space-time contour by the spin-1/2 particle. By contrast, all other approaches to worldline descriptions have adopted the second order form for the Dirac operator which forces a, conventional type, spin-dependent term of the form $\sigma_{\mu\nu}F^{\mu\nu}$ in the worldline action. Our first undertaking, in this paper, is to establish a direct connection between Polyakov's geometric spin-factor and the "dynamical" $\sigma \cdot F$ term.

A second consequence resulting from our approach is its adaptation to configurations involving *open* worldline contours. In physical terms this means that we are in a position to extend our considerations to processes wherein spin-1/2 entities enter as *external* particle states.[2] By contrast, Strassler's applications pertain to n-point functions with gauge field modes as external states which attach themselves to a fermionic loop [7]. The major part of our present effort is to extend worldline computational algorithms, hitherto provided for diagrams involving fermionic loops, to ones wherein *open* fermionic paths make their entrance. A relevant attempt, along the lines of Fradkin's approach [1], appears in [16] *albeit* falling short of explicitly dealing with γ-matrix manipulations.

[1] In fact, the characterization "worldline" strictly applies to the resulting structure which, through this integration, formulates itself in coordinate space.

[2] It is in this context that common grounds have been established with the eikonal-based work of Fried [4]–[6], who employs Schwinger's functional calculus as methodological tool and explicitly accommodates spin through the $\sigma \cdot F$ term.

Our ultimate hope is that, through the present work, the elegant separation between purely geometrical properties of matter particle paths, on the one hand, and dynamics, on the other, will emerge as one of the most potent aspects of the worldline casting of gauge field theories. Leaving the elucidation of this remark to an a posteriori discussion, let us proceed to exhibit the organization of the present paper. In the next section we display worldline expressions which will form the basis of our subsequent considerations. We shall proceed, in section 3, to make the connection between the "dynamical" $\sigma \cdot F$ term, appearing in the worldline action as a result of adopting the second order form for the Dirac operator and the "geometrical" Polyakov's spin factor. This is accomplished via a fortuitous application of a functional derivative operator first introduced in [17, 18]. Section 4 constitutes a central part of our paper. Specifically, we undertake the task of carrying out the procedure that will lead to an explicit worldline formula for the fermionic Green's function to second order in its perturbative expansion. A major aspect of this attempt is to present an expression which employs super-particle coordinates, i.e. a path integral description which uses both bosonic and fermionic variables. We proceed, in section 5, to discuss computational tools which are involved in practical applications of the worldlike scheme. A first such undertaking is carried out in section 6 where we establish, to second order, the equivalence between the Feynman diagrammatic and worldline perturbative expansions of the fermionic propagator in a non-abelian gauge theory. A direct one loop calculation is performed in section 7; it produces a final expression in parametrized form which is ready to enter perturbative estimates of higher order Green's functions with external fermion (as well as gauge field) legs. Our concluding remarks are presented in section 8, while some technical matters pertaining to the work in section 7 are displayed in an appendix.

2. Basic worldline expressions

In this section we shall discuss worldline formulas placing emphasis on fermionic Green's functions in a background of, non-abelian in general, gauge fields. We shall work in euclidean space-time in the context of which the spin-factor can be defined. Special emphasis will be placed on the second order Dirac formalism as our immediate aim (cf. next section) is to establish its connection with the spin-factor.

Let us commence our considerations with a short presentation of previous results stemming from the employment of the first order Dirac operator. A systematic procedure which takes one from the original field theoretical casting of the system to worldline formulas can be found in [9]. Here we shall proceed formally. Consider, first, the effective action functional, with dependence on the background gauge field. Its formal expression is

$$W[A] = \ln \mathrm{Det}[i\gamma \cdot D - (m - i\epsilon)]\,, \qquad D_\mu = \partial_\mu + igA_\mu\,. \tag{2.1}$$

Using Schwinger's proper time representation [19] and going to euclidean space-time we write

$$W[A] = -\operatorname{Tr}\int_0^\infty \frac{dT}{T} e^{-T(-\gamma\cdot D+m)}\,. \tag{2.2}$$

Computation of the non-Dirac γ matrix as well as non-color part of the trace in coordinate space representation leads, after a suitable choice of gauge,[3] to the worldline result [9]

$$W[A] = -\int_0^\infty \frac{dT}{T} e^{-Tm} \int_{x(0)=x(T)} \mathcal{D}x(\tau)\delta[\dot{x}(\tau)^2 - \text{const.}]\, \mathrm{tr}\Phi(C)\, \mathrm{Tr}_c P \times \\ \times \exp\left[ig\int_0^T \dot{x}\cdot A(x(\tau))\right], \tag{2.3}$$

where $\mathrm{tr}(\mathrm{Tr}_c)$ denotes trace over γ-matrix (color), P stands for path ordering and where the *spin-factor* $\Phi(C)$ [15] is given by

$$\Phi(C) = P\exp\left\{ \frac{1}{8}\int_0^T d\tau\, \omega_{\mu\nu}[x(\tau)][\gamma_\mu,\gamma_\nu]\right\}, \tag{2.4}$$

with

$$\omega_{\mu\nu}(x(\tau)) = \frac{1}{2}[\ddot{x}_\mu(\tau)\dot{x}_\nu(\tau) - \dot{x}_\mu(\tau)\ddot{x}_\nu(\tau)]\,, \qquad \dot{x}^2 = 1\,, \tag{2.5}$$

describing the orientation tensor of the local, perpendicular to the path, plane. Note that $|w|^2 \sim |\ddot{x}|^2$, which means that it furnishes the curvature when $|\dot{x}| = \text{const}$.

As already mentioned, our main preoccupation in this paper will be with fermionic Green's functions (propagators) whose definition involves open paths. To begin, let us discuss worldline expressions for the fermionic Green's function, in a (non-abelian) gauge field background. We first consider the case where the first order form for the Dirac operator is adopted. The corresponding Schwinger proper time formula for the Green's operator is

$$iG[A] = -\int_0^\infty dT\, e^{-T(-\gamma\cdot D+m)}\,. \tag{2.6}$$

Its worldline representation, in phase-space, reads

$$iG(x,y|A) = \int \mathcal{D}e(\tau)\int \mathcal{D}h(\tau)\int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau)\int \mathcal{D}p(\tau)\exp\left[i\int_0^1 d\tau h(\tau)\dot{e}(\tau)\right] \times \\ \times \exp\left[i\int_0^1 d\tau p(\tau)\cdot\dot{x}(\tau)\right] P\exp\left[-\int_0^1 d\tau e(\tau)\{m + i\gamma\cdot p(\tau)\}\right] \times \\ \times P\exp\left[ig\int_0^1 A(x(\tau))\cdot\dot{x}(\tau)\right]. \tag{2.7}$$

The integral over $\mathcal{D}h(\tau)$ on the right hand side supplies our commitment to a choice of gauge with respect to the reparametrization invariance requirement. The latter,

[3] We are here referring to a gauge choice with respect to parametrization invariance and mass identification. More specific comments on this issue will be made shortly.

corresponds to the transformation $\tau \to t = f(\tau)$, $\dot{f}(\tau) > 0$, $f(0) = 0$, $f(1) = 1$, under which we have $e(\tau) \to \dot{f}(\tau)e(t)$, $\dot{x}(\tau) \to \dot{f}(\tau)\dot{x}(t)$ and $h(\tau) \to \frac{1}{\dot{f}(\tau)}h(t)$.

The second order formalism for the Dirac operator, on the other hand, bases itself on the operator expression

$$iG[A] = (\gamma \cdot D + m) \int_0^\infty dT \, e^{-T[m^2 - (\gamma \cdot D)^2]}, \tag{2.8}$$

whose phase-space worldline representation turns out to be

$$\begin{aligned} iG(x,y|A) = & \int \mathcal{D}e(\tau) \int \mathcal{D}h(\tau) \int_{\substack{x(0)=x \\ x(1)=y}} \mathcal{D}x(\tau) \int \mathcal{D}p(\tau) \left[m - \frac{1}{2e(1)} \gamma \cdot \dot{x}(1) \right] \times \\ & \times \exp\left[i \int_0^1 d\tau h(\tau)\dot{e}(\tau) \right] \exp\left[i \int_0^1 d\tau p(\tau) \cdot \dot{x}(\tau) \right] \times \\ & \times \exp\left[- \int_0^1 d\tau e(\tau) \{ p^2(\tau) + m^2 \} \right] \times \\ & \times P \exp\left[ig \int_0^1 A(x(\tau)) \cdot \dot{x}(\tau) + \frac{1}{2} g \int_0^1 d\tau e(\tau) \sigma \cdot F(x(\tau)) \right]. \end{aligned} \tag{2.9}$$

As already stated our first preoccupation in this paper is to establish that the $\sigma \cdot F$ term in the above formula translates into the analogue of the spin-factor for *open* fermionic paths. The first step in this direction is to carry out the integration over $\mathcal{D}p(\tau)$.

We obtain

$$\begin{aligned} iG(x,y|A) = & \int \mathcal{D}e(\tau) \int \mathcal{D}h(\tau) \exp\left[i \int_0^1 d\tau h(\tau)\dot{e}(\tau) \right] C(e) \times \\ & \times \int_{\substack{x(0)=x \\ x(1)=y}} \mathcal{D}x(\tau) \left[m - \frac{1}{2e(1)} \gamma \cdot \dot{x}(1) \right] \times \\ & \times \exp\left[-\frac{1}{4} \int_0^1 d\tau \frac{\dot{x}^2(\tau)}{e(\tau)} - m^2 \int_0^1 d\tau e(\tau) \right] \times \\ & \times P \exp\left[ig \int_0^1 A(x(\tau)) \cdot \dot{x}(\tau) + \frac{1}{2} g \int_0^1 d\tau e(\tau) \sigma \cdot F(x(\tau)) \right], \end{aligned} \tag{2.10}$$

where

$$C(e) = \prod_{n=1}^{N} \left(\frac{N}{4\pi e_n} \right)^{D/2} \tag{2.11}$$

can be looked upon as a normalization factor.

Our immediate objective is to recast the above expression into a form where the open-line analogue of the spin-factor replaces the $\sigma \cdot F$ term. The relevant work will be carried out in the following section.

3. Spin-factor in the second order formalism

We commence our considerations which will lead to the recasting of the $\sigma \cdot F$ term in (2.9) by utilizing a functional derivative operator acting on path dependent quantities. For closed paths it has the meaning of an area derivative. For practical purposes we choose to follow Polyakov's proposal [17]. Migdal's definition [18], on the other hand, is based on geometrical requirements. Let us, then, introduce the operator by

$$\frac{\delta}{\delta s_{\mu\nu}(t)} \equiv \lim_{\varepsilon \to 0} \int_{-\varepsilon}^{+\varepsilon} d\sigma \, \sigma \frac{\delta^2}{\delta x_\mu(t+\sigma/2)\delta x_\nu(t-\sigma/2)} \,. \tag{3.1}$$

Consider the action of the above operator on the functional

$$I[\dot{x}(\tau)] \equiv \exp\left[-\frac{1}{4}\int_0^1 d\tau \frac{\dot{x}^2(\tau)}{e(\tau)}\right], \tag{3.2}$$

which leads to

$$\frac{\delta}{\delta s_{\mu\nu}(t)} \exp\left[-\frac{1}{4}\int_0^1 d\tau \frac{\dot{x}^2(\tau)}{e(\tau)}\right] = \frac{1}{2}\omega_{\mu\nu}(x(t))\left[-\frac{1}{4}\int_0^1 d\tau \frac{\dot{x}^2(\tau)}{e(\tau)}\right], \tag{3.3}$$

with $\omega_{\mu\nu}$ now given by[4]

$$\omega_{\mu\nu}(x(\tau)) = \frac{1}{2e^2(\tau)}[\ddot{x}_\mu(\tau)\dot{x}_\nu(\tau) - \dot{x}_\mu(\tau)\ddot{x}_\nu(\tau)] \,. \tag{3.4}$$

On the other hand, we find

$$\begin{aligned} &P\exp\left[ig\int_0^1 A(x(\tau))\cdot\dot{x}(\tau) + \frac{1}{2}g\int_0^1 d\tau e(\tau)\sigma\cdot F(x(\tau))\right] = \\ &= P\exp\left[\frac{i}{2}\int_0^1 d\tau e(\tau)\,\sigma\cdot\frac{\delta}{\delta s(\tau)}\right] P\exp\left[ig\int_0^1 d\tau \dot{x}(\tau)\cdot A(x(\tau))\right]. \end{aligned} \tag{3.5}$$

Inserting into (2.10) we readily derive, after partial integration,

$$\begin{aligned} iG(x,y|A) = &\int \mathcal{D}e(\tau)\int \mathcal{D}h(\tau)\exp\left[i\int_0^1 d\tau h(\tau)\dot{e}(\tau)\right] C(e)\,\times \\ &\times \int_{\substack{x(0)=x\\x(1)=y}} \mathcal{D}x(\tau)\left[m - \frac{1}{2e(1)}\gamma\cdot\dot{x}(1)\right]\times \\ &\times\exp\left[-\frac{1}{4}\int_0^1 d\tau\frac{\dot{x}^2(\tau)}{e(\tau)} - m^2\int_0^1 d\tau e(\tau)\right]\times \\ &\times P\exp\left[\frac{i}{4}\int_0^1 d\tau\,\sigma\cdot\omega(x(\tau))\right] P\exp\left[ig\int_0^1 d\tau\dot{x}(\tau)\cdot A(x(\tau))\right]. \end{aligned} \tag{3.6}$$

[4] Note that the expression that follows coincides with eq. (2.5) when $e^2(\tau) = 1$, which corresponds to a specific choice of gauge.

Even though the term 'spin-factor' per se pertains, as a geometrical quantity, to closed paths, we shall employ, from hereon, this nomenclature for the expression given by eq. (2.4) even when it enters quantities defined on open paths, such as the above.

The same considerations applied to the effective action functional whose formal expression, in the second order formalism, is

$$-W[A] = \frac{1}{2}\,\mathrm{Tr}\int_0^\infty \frac{dT}{T} e^{-T(m^2-(\gamma\cdot D)^2)}\,, \tag{3.7}$$

leads to the worldline representation

$$\begin{aligned} -W[A] = &\int \mathcal{D}e(\tau)\int \mathcal{D}h(\tau)\exp\left[i\int_0^1 d\tau h(\tau)\dot{e}(\tau)\right]\bar{C}(e)\times \\ &\times\int_{x(0)=x(1)}\mathcal{D}x(\tau)\exp\left[-\frac{1}{4}\int_0^1 d\tau\frac{\dot{x}^2(\tau)}{e(\tau)} - m^2\int_0^1 d\tau e(\tau)\right]\times \\ &\times \mathrm{tr}P\exp\left[\frac{i}{4}\int_0^1 d\tau\,\sigma\cdot\omega(x(\tau))\right]\mathrm{Tr}_c P\exp\left[ig\int_0^1 d\tau\dot{x}(\tau)\cdot A(x(\tau))\right], \end{aligned} \tag{3.8}$$

where $\bar{C}(e)\equiv\frac{1}{e(1)}C(e)$.

For a proper choice of gauge the above result matches the one that has been obtained [9] through the first order formalism, cf. eq. (2.3). The notable accomplishment resulting from the manipulations that have just been carried out is an apparent dissociation of a geometrical feature (cf. spin-factor), characterizing the propagation of particle-like modes, from the dynamics operating in the system. The latter seem to be exclusively carried out by the Wilson factor $P\exp\left[ig\int_0^1 d\tau\dot{x}(\tau)\cdot A(x(\tau))\right]$. As will actually turn out, computations involving the spin factor term do, in an implicit manner, associate themselves with dynamical input.

Before proceeding further we shall commit ourselves to a choice of gauge in relation to reparametrization invariance. Designating as dynamical variable the proper time we set

$$t = \int_0^\tau d\tau'\, e(\tau'), \quad T = \int_0^1 d\tau\, e(\tau) \tag{3.9}$$

and obtain

$$\begin{aligned} iG(x,y|A) = &\int_0^\infty dT e^{-Tm^2}\int_{\substack{x(0)=x\\ x(T)=y}}\mathcal{D}x(\tau)\left[m-\frac{1}{2}\gamma\cdot\dot{x}(T)\right]\exp\left[-\frac{1}{4}\int_0^1 d\tau\dot{x}^2(\tau)\right]\times \\ &\times P\exp\left[\frac{i}{4}\int_0^T d\tau\,\sigma\cdot w(x(\tau))\right]P\exp\left[ig\int_0^T d\tau\dot{x}(\tau)\cdot A(x(\tau))\right], \end{aligned} \tag{3.10}$$

where the measure $\mathcal{D}x(\tau)$ is now defined as $\lim_{N\to\infty}(N/4\pi T)^{ND/2}\sum_{n=0}^N d^D x_n$, while $w_{\mu\nu}$ is, on account of our choice of gauge, given by

$$w_{\mu\nu}(x) = \frac{T}{2}(\ddot{x}_\mu\dot{x}_\nu - \dot{x}_\mu\ddot{x}_\nu)\,. \tag{3.11}$$

Turning our attention to the full fermionic Green's function we write

$$iG(x,y) = \int_0^\infty dT e^{-Tm^2} \int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau) \left[m - \frac{1}{2}\gamma \cdot \dot{x}(T)\right] \exp\left[-\frac{1}{4}\int_0^1 d\tau \dot{x}^2(\tau)\right] \times$$
$$\times P\exp\left[\frac{i}{4}\int_0^T d\tau\, \sigma \cdot w(x(\tau))\right] \left\langle P\exp\left[ig\int_0^T d\tau \dot{x}(\tau)\cdot A(x(\tau))\right]\right\rangle_A, \quad (3.12)$$

where the expectation value in the gauge field sector includes gauge fixing terms, ghost integration and Dirac determinant contribution.

Generalizing, we infer that, in the worldline language, quantities of theoretical interest, such as n-point Green's functions and generating functionals, draw all dynamical input from expectation values of Wilson lines or loops. Such objects have been extensively studied in the past, see, for example, [17] and [20]–[24]. Any possible non-perturbative application of the present formalism will, therefore, be greatly facilitated by relevant, accumulated knowledge.

Even though eq. (3.12) will serve as the starting point for our subsequent considerations, we find it useful to recast it in a form which adopts particle-based coordinates. Along with practical consequences, this will facilitate a direct comparison with Strassler's (closed path) worldline expressions. We shall devote the remainder of the present section to this task.

As a first step, let us cast (3.12) in the form $iG(x,y) = a(u^2,m^2)u\cdot\gamma + b(u^2,m^2)$, $u_\mu \equiv \frac{1}{T}(y-x)_\mu$ by writing

$$iG(x,y) = \frac{1}{D}\int_0^\infty dT e^{-Tm^2} \int_{\substack{x(0)=x\\ x(1)=y}} \mathcal{D}x(\tau) \times$$
$$\times \left\{\left[\frac{m}{D}\delta_{\alpha\beta} - \frac{1}{2}\gamma_\alpha \dot{x}_\beta(T)\right] \mathrm{tr}\left[\gamma_\alpha\gamma_\beta P\exp\left[\frac{i}{4}\int_0^T d\tau\, \sigma\cdot w(x(\tau))\right]\right]\right\} \times$$
$$\times \left\langle P\exp\left[ig\int_0^T d\tau \dot{x}(\tau)\cdot A(x(\tau))\right]\right\rangle_A, \quad (3.13)$$

which follows via the identifications $b = \frac{1}{D}\,\mathrm{tr}(iG)$, $au_\mu = \frac{1}{D}\,\mathrm{tr}(\gamma_\mu iG)$ and once taking into account that $trI = D$.

We now use the identity

$$\mathrm{tr}\left\{\gamma_\alpha\gamma_\beta P\exp\left[\frac{i}{4}\int_0^T d\tau\, \sigma\cdot w(x(\tau))\right]\right\} = -\int_{\psi(0)+\psi(T)=0}[d\psi]2\psi_\alpha(0)\psi_\beta(T) \times$$
$$\times \exp\left[-\frac{1}{2}\int_0^T d\tau \psi\cdot\dot{\psi} - \frac{1}{2}\int_0^T d\tau \psi_\mu(\tau)\psi_\nu(\tau)w_{\mu\nu}(x(\tau))\right], \quad (3.14)$$

where the ψ_μ are Grasmann variables whose correlator, with respect to the action $\left[-\frac{1}{2}\int_0^T d\tau\psi\cdot\dot{\psi}\right]$, is determined by taking into consideration that

$$\{\psi_\mu, \psi_\nu\} = \delta_{\mu\nu} \quad (3.15)$$

and

$$\frac{1}{D}\int_{\psi(0)+\psi(T)=0}[d\psi]\exp\left[-\frac{1}{2}\int_0^T d\tau\psi\cdot\dot{\psi}\right]=1\,. \tag{3.16}$$

We obtain

$$\langle\psi_\mu(\tau_1)\psi_\nu(\tau_2)\rangle_\psi=\frac{1}{2}\delta_{\mu\nu}\,\mathrm{sign}(\tau_1-\tau_2)\,. \tag{3.17}$$

One is thereby led to the following particle-based, to be referred to as 'super-particle', representation for the full fermionic propagator

$$\begin{aligned}iG(x,y)=&-\frac{1}{D}\int_0^\infty dTe^{-Tm^2}\int_{\substack{x(0)=x\\x(1)=y}}\mathcal{D}x(\tau)\left[\frac{m}{D}\delta_{\alpha\beta}-\frac{1}{2}\gamma_\alpha\dot{x}_\beta(T)\right]\int_{\psi(0)+\psi(T)=0}[d\psi]\times\\&\times 2\psi_\alpha(0)\psi_\beta(T)\exp\left[-\frac{1}{4}\int_0^T d\tau\dot{x}^2-\frac{1}{2}\int_0^T d\tau\psi\cdot\dot{\psi}-\frac{1}{2}\int_0^T d\tau\psi_\mu\psi_\nu w_{\mu\nu}[x(\tau)]\right]\times\\&\times\left\langle P\exp\left[ig\int_0^T d\tau\dot{x}(\tau)\cdot A(x(\tau))\right]\right\rangle_A\,.\end{aligned} \tag{3.18}$$

This is an essentially new result which not only extends Strassler's expressions to *open* fermionic lines but encodes spin effects through a *geometrical quantity* entering a particle-based action functional. At the same time it underlines the role of the Wilson line operator as the sole agent of the dynamics operating in the system. The spin factor, in other words, has joined the rest of the path-dependent (super) coordinates to account for particle-based characteristics of spin-1/2 matter field propagation. Generalizations of spin factor expressions pertaining to higher spins have been discussed, from a different viewpoint, in ref. [25]. The bottom line is that the spin factor accounts for geometrical features of paths induced on them by the nature of the particle-like entity which traverses them.

Mention should be made, at this point, of the attempt in ref. [26] to tackle the problem of open line propagation of spin-1/2 particles in a gauge field background. The relevant methodology has a different philosophy (analytical rather than geometrical) and restricts itself to situation wherein the gauge field(s) do not acquire a dynamical character, i.e. they remain external. By staying faithful to Polyakov's geometric point of view [15], on the other hand, we have both attained simplicity in form of worldline expressions while achieving, at the same time, extensions beyond the free particle case.

The corresponding expression for the effective action functional is

$$\begin{aligned}-W[A]=&\frac{1}{2}\int_{\psi(0)+\psi(T)=0}[d\psi]\int_0^\infty dTe^{-Tm^2}\int_{\substack{x(0)=x\\x(T)=y}}\mathcal{D}x(\tau)\times\\&\times\exp\left[-\frac{1}{4}\int_0^T d\tau\dot{x}^2-\frac{1}{2}\int_0^T d\tau\psi\cdot\dot{\psi}-\frac{1}{2}\int_0^T d\tau\psi_\mu\psi_\nu w_{\mu\nu}(x(\tau))\right]\times\\&\times\mathrm{Tr}_c P\exp\left[ig\int_0^T d\tau\dot{x}(\tau)\cdot A(x(\tau))\right]\,.\end{aligned} \tag{3.19}$$

The above equation corresponds to Strassler's result, modulo the presence of the spin-factor term in place of $\sigma \cdot F$. It serves as the starting point for computational rules involving quark loops. The relevant particle-based action can be read off the above formulas and has as follows

$$S = \frac{1}{4}\int_0^T d\tau \dot{x}^2 + \frac{1}{2}\int_0^T d\tau \psi \cdot \dot{\psi} + \frac{1}{2}\int_0^T d\tau \psi_\mu \psi_\nu w_{\mu\nu}(x(\tau)) - ig\int_0^T d\tau \bar{z}\dot{x} \cdot A\, z\,, \quad (3.20)$$

where the z-fields serve to enforce path ordering when background fields are present.

Note that under the supersymmetric transformation $\delta x_\mu = -i\theta\dot{\psi}_\mu$, $\delta\psi_\mu = \theta\dot{x}_\mu$ the first two terms in (3.20) remain invariant whilst the last two do not. On the other hand, we can easily retrieve the $\sigma \cdot F$ term, by partial integration, from the spin-factor. This brings us right back to the Strassler form for the worldline action which *is* supersymmetric invariant.[5] We shall refer to (3.20) as super-worldline particle action.

4. Analytic manipulations with the spin factor: fermionic propagator to 2^{nd} order

Besides its interpretational appeal as a geometrical agent that accounts for the spin of a propagating particle, the spin factor presents practical advantages as it actually expedites analytic manipulations involved in the computation of fermionic Green's functions and effective action functionals. Samples of the latter case, a most notable of which is the (non-perturbative) extraction of the Chern-Simons action functional in three dimensions, have been given in [9]. Here we focus our efforts on calculational aspects surrounding fermionic propagators.

4.1 General considerations

In this subsection we shall conduct intuitive type of manipulations with the spin factor in an attempt to gain practical familiarity with the way it should be handled. Full justification of our resulting expressions will come later on.

Suppose we were to expand the relevant exponential which furnishes the spin factor. Starting from the bosonic sector, we would find ourselves having to compute expectation values of the type $\langle(\dot{x}_\mu(\tau)\ddot{x}_\nu(\tau) - \ddot{x}_\mu(\tau)\dot{x}_\nu(\tau))\cdots\rangle_x$. Clearly, the only way to get a non-null result is for a four-vector, say k_μ, to make its entrance so that, in combination with the four-vector $(x-y)_\mu$ which will emerge from the path integration, would facilitate the creation of antisymmetric combinations. Equivalently, unless the first derivative is discontinuous at some $\tau = \tau_i$ the aforementioned expectation value terms, once recast the form $\langle(\dot{x}_\mu(\tau)\dot{x}_\nu(\tau+\alpha) - \dot{x}_\mu(\tau)\dot{x}_\nu(\tau-\alpha))\cdots\rangle_x$,

[5]Note that supersymmetry appears to be manifestly broken in (3.20) due to the boundary conditions involved in the partial integration which produced the spin factor.

($\alpha \to 0$), would lead to a vanishing result except at points where a 'force' acts, thereby 'injecting' a four-momentum k_μ on the fermionic line.

Consider the situation where M such points τ_i are present on a given fermionic line. Let us set, as $\epsilon \to 0$,

$$\frac{1}{2}[\dot{x}_\mu(\tau_i+\epsilon) - \dot{x}_\mu(\tau_i-\epsilon)] = ik_{i\mu}\,, \qquad i = 1, \ldots, M\,, \tag{4.1}$$

or, equivalently,

$$\ddot{x}_\mu(\tau) \longrightarrow 2i\sum_{i=1}^{M} k_{i\mu}\delta(\tau-\tau_i)\,. \tag{4.2}$$

We surmise (here C denotes an open curve)

$$\begin{aligned}\Phi(C) \to \lim_{\epsilon\to 0}\exp\left[\frac{T}{4}\sum_{i=1}^{M}\int_{\tau_i-\epsilon}^{\tau_i+\epsilon} d\tau\psi_\mu(\tau)\psi_\nu(\tau)[\ddot{x}_\mu(\tau)\dot{x}_\nu(\tau) - \ddot{x}_\nu(\tau)\dot{x}_\mu(\tau)\right] = \\ = \exp -\frac{T}{2}\sum_{i=1}^{M}\psi_\mu(\tau_i)\psi_\nu(\tau_i)[ik_{i\mu}\dot{x}_\nu(\tau_i) - ik_{i\nu}\dot{x}_\mu(\tau_i)]\,.\end{aligned} \tag{4.3}$$

Momentum conservation can be taken into account by setting $\sum_{i=1}^{M} k_{i\mu} = 0$.

Clearly, M measures the number of points where a gauge field is applied on a given fermionic path.[6] To expedite our upcoming manipulations of the spin factor expression given by (4.3) we adopt, throughout, the following prescription for the equal time worldline fermionic correlator

$$\langle\psi_\mu(\tau)\psi_\nu(\tau)\rangle_\psi = \frac{1}{2}\delta_{\mu\nu}\,\mathrm{sign}(0) = 0\,. \tag{4.4}$$

One promptly surmises that any term in the expansion of order greater than M will yield a vanishing contribution to the propagator. So, not only is M associated with perturbative order but also sets a bound on the contributing terms in the expansion of the spin factor.

Armed with the above concrete as well as of practical value realizations, we are in position to turn our attention to the 2nd order expression of the fermionic Green's function and gain specific insights with respect to spin factor contributions to its worldline form. This is precisely the task that we shall carry out in the next subsection.

4.2 Spin factor contribution to the second order fermionic Green's function

Expanding the Wilson exponential in (3.18) we bring down, to first order in g^2, the factor $\int_0^T d\tau_1 \int_0^T d\tau_2 \dot{x}_\mu(\tau_2)\dot{x}_\nu(\tau_1)\langle A_\mu(x(\tau_2))A_\nu(x(\tau_1))\rangle_A$. Working in the Feynman

[6] It is in this context that an earlier comment, to the effect that the computation of the spin-factor expression requires dynamical input, was made.

gauge we write

$$
\begin{aligned}
iG^{(2)}(x,y) = g^2 \frac{1}{D} c_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \int_0^\infty dT e^{-Tm^2} \int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2-\tau_1) \int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau) \times \\
\times \dot{x}(\tau_2)\cdot\dot{x}(\tau_1) \left[\frac{m}{D}\delta_{\alpha\beta} - \frac{1}{2}\gamma_\alpha \dot{x}_\beta(T)\right] \int_{\psi(0)+\psi(T)=0} [d\psi(\tau)] 2\psi_\alpha(0)\psi_\beta(T) \times \\
\times \exp\left[-\frac{1}{4}\int_0^T d\tau \dot{x}^2 - \frac{1}{2}\int_0^T d\tau \psi\cdot\dot{\psi} - \right. \\
\left. -\frac{1}{2}\int_0^T d\tau \psi_\mu\psi_\nu w_{\mu\nu} - ik\cdot(x(\tau_2)-x(\tau_1))\right]. \qquad (4.5)
\end{aligned}
$$

Focusing our attention on the spin-factor expression, let us consider the first order term, to be denoted by $\Phi^{(1)}$, in its expansion. According to the general discussion of the previous subsection we determine

$$
\Phi^{(1)} = \frac{T}{2}\sum_{i=1,2} \psi_\mu(\tau_i)\psi_\nu(\tau_i)[ik_{\mu i}\dot{x}_\nu(\tau_i) - ik_{\nu i}\dot{x}_\mu(\tau_i)]\,. \qquad (4.6)
$$

Setting $k_2 = -k_1 = k$ we obtain

$$
\begin{aligned}
\Phi^{(1)} &= \frac{T}{2}\psi_\mu(\tau_2)\psi_\nu(\tau_2)[ik_\mu\dot{x}_\nu(\tau_1) - ik_\nu\dot{x}_\mu(\tau_1)] - \\
&- \frac{T}{2}\psi_\mu(\tau_1)\psi_\nu(\tau_1)[ik_\mu\dot{x}_\nu(\tau_2) - ik_\nu\dot{x}_\mu(\tau_2)]\,. \qquad (4.7)
\end{aligned}
$$

Consider, next, the path integration over the ψ fields. According to (4.3) the contribution from the above terms is zero. Had our computation referred to the effective action functional $W[A]$ this would be the end of the story, as far as path integration over the fermionic coordinates is concerned. For the Green's function, on the other hand, a four-spinor factor also appears for which we determine (we keep only the antisymmetric contribution, under the exchange $\mu \leftrightarrow \nu$)

$$
2\langle\psi_\alpha(0)\psi_\beta(T)\psi_\mu(\tau_i)\psi_\nu(\tau_i)\rangle_\psi = \frac{1}{2}[\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}]\,, \qquad i=1,2\,. \qquad (4.8)
$$

Reserving the same notation for the quantity that will result after the path integration with respect to the particle coordinates we write

$$
\Phi^{(1)} \to \frac{T}{2} ik_\alpha[\dot{x}_\beta(\tau_2) - \dot{x}_\beta(\tau_1)] - \frac{T}{2} ik_\beta[\dot{x}_\alpha(\tau_2) - \dot{x}_\alpha(\tau_1)]\,, \qquad (4.9)
$$

where the arrow serves to signify the fact that a functional integration with respect to the $x_\mu(\tau)$ fields remains to be carried out. Now, the full computation of $\Phi^{(1)}$ will encounter divergencies the occurence of which can be seen by referring to eq. (2.3). There we see that integration over (open or closed) gives a null result unless $|\dot{x}|^2 = \text{const.}$, in which case we have a delta function type of divergence. We regularize according to [17]

$$\langle \dot{x}_\rho(\tau)\dot{x}_\sigma(\tau)\rangle_x \to \lim_{\epsilon\to 0} -\frac{1}{T}\delta_{\rho\sigma}\int_{-\epsilon}^{+\epsilon} d\sigma \frac{\delta^2}{\delta J_\rho(\tau+\sigma/2)\delta J_\sigma(\tau-\sigma/2)} \left\langle e^{i\int_0^T d\tau J(\tau)\cdot\dot{x}(\tau)} \right\rangle_x \Big|_{J=0}, \tag{4.10}$$

which will eventually lead to the result, intimately connected with reparametrization invariance,

$$\langle \dot{x}_\rho(\tau)\dot{x}_\sigma(\tau)\rangle_x = \frac{2}{T}\delta_{\rho\sigma}\,. \tag{4.11}$$

Assuming the validity of (4.11) we can now replace (4.9) by

$$\Phi^{(1)} \to ik_\alpha[\dot{x}_\beta(\tau_1) - \dot{x}_\beta(\tau_2)] - ik_\beta[\dot{x}_\alpha(\tau_1) - \dot{x}_\alpha(\tau_2)]\,. \tag{4.12}$$

Next, we consider the contribution from the second order term which, according to our general discussion, is given by

$$\begin{aligned}\Phi^{(2)} = {}& \frac{T^2}{8}\psi_\mu(\tau_2)\psi_\nu(\tau_2)\psi_\kappa(\tau_1)\psi_\lambda(\tau_1)\,\times \\ &\times [k_\mu\dot{x}_\nu(\tau_2) - k_\nu\dot{x}_\mu(\tau_2)][k_\kappa\dot{x}_\lambda(\tau_1) - k_\lambda\dot{x}_\kappa(\tau_1)] + (1\leftrightarrow 2)\,.\end{aligned} \tag{4.13}$$

Taking on the task of calculating expectation values in the ψ-sector of the (super)particle system we determine, once keeping only the symmetric term under the exchange $1\leftrightarrow 2$,

$$2\langle\psi_\alpha(0)\psi_\beta(T)\psi_\mu(\tau_2)\psi_\nu(\tau_2)\psi_\kappa(\tau_1)\psi_\lambda(\tau_1)\rangle_\psi = -\frac{1}{4}\delta_{\alpha\beta}[\delta_{\mu\lambda}\delta_{\nu\kappa} - \delta_{\nu\lambda}\delta_{\mu\kappa}][\text{sign}(\tau_1-\tau_2)]^2\,. \tag{4.14}$$

As before, we keep the notation $\Phi^{(2)}$ for the quantity which results from path integration in the super-particle sector. Substitution of (4.14) into (4.13) furnishes the intermediate result

$$\begin{aligned}\Phi^{(2)} \to {}& +\frac{T^2}{8}\delta_{\alpha\beta}[k_\mu\dot{x}_\nu(\tau_2) - k_\nu\dot{x}_\mu(\tau_2)][k_\mu\dot{x}_\nu(\tau_1) - k_\nu\dot{x}_\mu(\tau_1)]G_F^2(\tau_1,\tau_2) = \\ &= \frac{T^2}{4}\delta_{\alpha\beta}(k^2\delta_{\mu\nu} - k_\mu k_\nu)\dot{x}_\mu(\tau_1)\dot{x}_\nu(\tau_2)G_F^2(\tau_1,\tau_2)\,,\end{aligned} \tag{4.15}$$

where we have, following Strassler, set $G_F(\tau_1,\tau_2) = \text{sign}(\tau_1-\tau_2)$.[7]

Further utilization of the bosonic sector path integration allows us to cast $\Phi^{(2)}$ in the more simplified form

$$\Phi^{(2)} \to -\delta_{\alpha\beta}(k^2\delta_{\mu\nu} - k_\mu k_\nu)\delta_{\mu\nu}G_F^2(\tau_1,\tau_2) = -\delta_{\alpha\beta}(D-1)k^2G_F^2(\tau_1,\tau_2)\,. \tag{4.16}$$

[7]The reason for keeping the factor $G_F^2(\tau_1,\tau_2)$ in eq. (4.15), given that it is equal to unity, is because we wish to explicitly display the common features with Strassler's closed fermionic path expressions.

All higher order terms in the exponential expansion of the spin-factor term give vanishing contribution to the Green's function. Here, we explicitly witness the harmonization of the spin-factor with dynamical aspects of the calculation: Recognizing the order of the perturbative expansion the spin-factor series terminates when the number of vertices (points on the path where a 'force' is exerted) is saturated.

Putting together everything that was so far carried out, we arrive at the following worldline expression for the fermionic propagator, to second order in perturbation theory,

$$
\begin{aligned}
iG^{(2)}(x,y) = -g^2 c_F \int \frac{d^D k}{(2\pi)^D}\frac{1}{k^2}\int_0^\infty dT e^{-Tm^2}\int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2-\tau_1)\int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau)\,\times \\
\times\left[\frac{m}{D}\delta_{\alpha\beta}-\frac{1}{2}\gamma_\alpha \dot{x}_\beta(\tau)\right]\Big\{\delta_{\alpha\beta}\dot{x}(\tau_2)\cdot\dot{x}(\tau_1)+ik_\beta[\dot{x}_\alpha(\tau_1)-\dot{x}_\alpha(\tau_2)]- \\
-ik_\alpha[\dot{x}_\beta(\tau_1)-\dot{x}_\beta(\tau_2)]-\delta_{\alpha\beta}G_F^2(\tau_1,\tau_2)(D-1)k^2\Big\}\times \\
\times\exp\left[-\frac{1}{4}\int_0^T d\tau\,\dot{x}^2 - ik\cdot(x(\tau_1)-x(\tau_2))\right]. \qquad (4.17)
\end{aligned}
$$

The corresponding expression for the effective action functional $W[A]$ follows in an analogous way and reads

$$
\begin{aligned}
W[A] = \frac{D}{2}g^2 \int \frac{d^D k}{(2\pi)^D}\,\mathrm{Tr}_c A_\mu(k)A_\nu(-k)\int_0^\infty dT e^{-Tm^2}\int_0^T d\tau_1\int_0^T d\tau_2\theta(\tau_2-\tau_1)\,\times \\
\times\int_{x(0)=x(T)} \mathcal{D}x(\tau)[\dot{x}_\mu(\tau_1)\dot{x}_\nu(\tau_2)-G_F^2(\tau_1,\tau_2)(k^2\delta_{\mu\nu}-k_\mu k_\nu)\,\times \\
\times\exp\left[-\frac{1}{4}\int_0^T d\tau\,\dot{x}^2 - ik\cdot(x(\tau_1)-x(\tau_2))\right]. \qquad (4.18)
\end{aligned}
$$

The above results express the embodiment of spin factor contribution to worldline formulas. Clearly, the immediate question is whether (4.17) furnishes the same perturbative expression for the propagator that one extracts, conventionally, through Feynman diagrammatic, rather than worldline, reasoning. That this is, indeed, the case will be established in section 6. More interesting, of course, is the question regarding whether the formalism that has been developed in this work can define autonomous approaches to field theoretical computations or, at least, offer practical advantages for higher order perturbative calculations for processes involving external fermionic fields. Setting this issue aside we shall, in the next section, turn our attention to the establishment of a set of rules pertaining to expectation values of bosonic 'field' products.

5. Computational tools in the super-particle representation

The novel feature of the worldline formalism is the super-particle mode of description which pervades its expressions for each given quantity of physical interest. Our spin factor manipulations in the previous section have mainly concentrated on aspects surrounding path integration over fermionic coordinates. In the present section we shall conduct a systematic discussion of the particle-based (super)sector whose ultimate aim is to promote the practical side of the worldline scheme.

Focusing on the bosonic sector of super-particle actions we realize that its kinetic term has the conventional form $\int_0^T d\tau \dot{x}^2(\tau)$. Corresponding propagators can be appropriately defined. For open lines the Green's function is denoted by $\Delta(\tau, \tau')$. It obeys the equation

$$\frac{\partial^2}{\partial \tau^2}\Delta(\tau,\tau')) = -\delta(\tau-\tau')\,, \tag{5.1}$$

with boundary conditions $\Delta(0,\tau') = \Delta(T,\tau') = 0$.

For closed contours the situation demands closer inspection. The relevant Green's function, to be denoted by $G_B(\tau,\tau')$, obeys an equation of the form

$$\frac{1}{2}\frac{\partial^2}{\partial \tau^2}G_B(\tau,\tau')) = \delta(\tau-\tau') - \frac{1}{T}\,, \tag{5.2}$$

where T is the length, according to a chosen parametrization, of the (closed) contour. The constant term in the above expression represents a uniform background charge density which is necessary to produce a finite potential for the compact configuration. Its choice is by no means unique; the above form coincides with Strassler's convention [7].

One obtains, respectively,

$$\Delta(\tau,\tau') = \frac{\tau(T-\tau')}{T}\theta(\tau'-\tau) + \frac{\tau'(T-\tau)}{T}\theta(\tau-\tau')\,, \qquad \frac{\partial}{\partial\tau}\Delta(\tau,\tau') = \theta(\tau'-\tau) - \frac{\tau'}{T} \tag{5.3}$$

and

$$G_B(\tau,\tau') = \frac{|\tau-\tau'|}{T}(T-|\tau-\tau'|)\,, \qquad \frac{\partial}{\partial\tau}G_B(\tau,\tau') = \mathrm{sign}(\tau-\tau') - \frac{2(\tau-\tau')}{T}\,. \tag{5.4}$$

We also mention the relations

$$\frac{\partial}{\partial\tau}[\Delta(\tau,\tau_2)-\Delta(\tau,\tau_1)] = \frac{1}{2}\frac{\partial}{\partial\tau}[G_B(\tau,\tau_1)-G_B(\tau,\tau_2)]\,, \qquad \frac{\partial^2}{\partial\tau\partial\tau'}\Delta(\tau,\tau') = \frac{1}{2}\ddot{G}_B(\tau,\tau'), \tag{5.5}$$

where the double dot on G_B signifies second derivative with respect to τ. Finally, during our calculations we shall employ the rules $\dot{G}_B(\tau,\tau) = 0$ and $\dot{G}_B^2(\tau,\tau) = 1$ which follow from our conventions. We shall also use the property $\dot{G}_B(\tau,\tau') = -\dot{G}_B(\tau',\tau)$.

Our first application is to establish the validity of eq. (4.11). Using (4.10) we readily determine

$$\langle \dot{x}_\rho(\tau)\dot{x}_\sigma(\tau)\rangle_x = \lim_{\epsilon\to 0}\frac{1}{T}\delta_{\rho\sigma}\int_{+\epsilon}^{-\epsilon} d\sigma \ddot{G}_B\left(\tau+\frac{\sigma}{2},\tau-\frac{\sigma}{2}\right) = \frac{2}{T}\delta_{\rho\sigma}\,. \tag{5.6}$$

One, thereby, explicitly verifies that integration over the bosonic coordinates $x(\tau)$ produces a result induced by parametrization invariance as it yields a constant measure for the velocity: $\langle|\dot{x}|^2\rangle = 2D/T$.

Next, we undertake the computation of expectation values of the first few products of derivatives of bosonic 'fields' $\dot{x}_\mu(\tau)$, with action functional the one that arose in connection with our second order fermionic propagator expression, cf. eq. (4.17). The particle-based (bosonic) action reads

$$S = \frac{1}{4}\int_0^T d\tau\dot{x}^2 + ik\cdot[x(\tau_2) - x(\tau_1)]\,. \tag{5.7}$$

The fact that terms of higher than 2^{nd} power are absent in S encourages the employment of techniques which base themselves on solutions of the classical equations of motion. The latter read

$$\ddot{x}^{cl}_\mu(\tau) = 2ik_\mu[\delta(\tau-\tau_2) - \delta(\tau-\tau_1)]\,. \tag{5.8}$$

with solution

$$x^{cl}_\mu(\tau) = \eta_\mu(\tau) - 2ik_\mu[\Delta(\tau,\tau_2) - \Delta(\tau,\tau_1)]\,, \tag{5.9}$$

where $\eta_\mu(\tau) = \frac{(y-x)_\mu}{T}\tau + x_\mu = u_\mu\tau + x_\mu$.

Under the substitution $x(\tau)\to x(\tau) + x^{cl}(\tau)$ the action becomes

$$S = S_{cl} + \frac{1}{4}\int_0^T d\tau\dot{x}^2(\tau)\,, \tag{5.10}$$

with

$$S_{cl} = \frac{1}{4}Tu^2 + ik\cdot u(\tau_2-\tau_1) + k^2G_B(\tau_1,\tau_2)\,, \tag{5.11}$$

Consider, first, the case of $\langle\dot{x}_\mu(\tau)\rangle$. We readily determine

$$\langle\dot{x}_\mu(\tau)\rangle = \mathcal{N}^{-1}\int_{\substack{x(0)=x\\x(T)=y}}\mathcal{D}x(\tau)\dot{x}_\mu(\tau)e^{-S} = \dot{x}^{cl}_\mu(\tau)\,, \tag{5.12}$$

where we have defined

$$\mathcal{N} = \int_{\substack{x(0)=x\\x(T)=y}}\mathcal{D}x(\tau)e^{-S}\,. \tag{5.13}$$

Note that

$$\begin{aligned}\dot{x}^{cl}_\mu(\tau) &= u_\mu - 2ik_\mu\frac{\partial}{\partial\tau}[\Delta(\tau,\tau_2) - \Delta(\tau,\tau_1)]\\ &= u_\mu - ik_\mu\left[\dot{G}_B(\tau,\tau_1) - \dot{G}_B(\tau,\tau_2)\right]\,.\end{aligned} \tag{5.14}$$

which, given the equal time rules mentioned after eq. (5.5) along with the fact that $\dot{G}_B(\tau,\tau') = -\dot{G}_B(\tau',\tau)$, yields

$$\dot{x}^{cl}_\mu(\tau_i) = u_\mu + ik_\mu\dot{G}_B(\tau_1,\tau_2)\,,\qquad i=1,2\,. \tag{5.15}$$

Observe, in passing, the consistency of the above result with substitution (4.2) associated with non-vanishing contributions of the spin factor in the bosonic sector. Indeed, we determine

$$\langle \dot{x}_\mu(\tau_1+\epsilon) - \dot{x}_\mu(\tau_1-\epsilon)\rangle_x = -ik_\mu\left[\dot{G}_B(\tau_1+\epsilon,\tau_1) - \dot{G}_B(\tau_1-\epsilon,\tau_1)\right] = -2ik_\mu = 2ik_{1\mu}\,. \tag{5.16}$$

For the correlator we find

$$\begin{aligned}\langle \dot{x}_\mu(\tau_1)\dot{x}_\mu(\tau_2)\rangle_x &= \dot{x}^{cl}_\mu(\tau_1)\dot{x}^{cl}_\mu(\tau_2) + \mathcal{N}_o\int_{x(0)=x(T)=0}\mathcal{D}x(\tau)\dot{x}_\mu(\tau_1)\dot{x}_\nu(\tau_2)e^{-\frac{1}{4}\int_0^T d\tau\dot{x}^2}\\ &= \dot{x}^{cl}_\mu(\tau_1)\dot{x}^{cl}_\mu(\tau_2) + \delta_{\mu\nu}\ddot{G}_B(\tau_1,\tau_2)\,,\end{aligned} \tag{5.17}$$

where $\mathcal{N}_o = 1/(4\pi T)^{D/2}$.

At this point we note that the quantity $\ddot{G}_B(\tau_i,\tau_j)$, which will make regular appearances into our expressions, contains a delta function component whose presence gives rise diagrams wherein two gluonic lines attach themselves to a fermionic (world)line at the same point. In order to conform with the, string-inspired, Bern-Kosower rules [14] we shall, following Strassler [7], systematically perform an integration by parts which replaces $\ddot{G}_B(\tau_i,\tau_j)$ by $\dot{G}^2_B(\tau_i,\tau_j)$. It should be remarked that such a practice introduces no ambiguities associated with the end point behavior of integrations with respect to the $d\tau_i$'s, where τ_i denotes a point of gauge field contact with the super-particle contour. As it turns out, partial integration invariably produces the combination $\dot{G}^2_B(\tau_i,\tau_j) - G^2_F(\tau_i,\tau_j)$ in the final formulas.[8]

In conclusion, our work in this section underlines the parallelism between open and closed line computational subtlelties. Nothing that arises in the former fails to do so in the latter case, or vice versa. The only difference is that additional terms in the super-particle action are induced by the new boundary conditions but which have no fundamental impact, as far as divergent behavior is concerned. Even though we shall not, in the present paper, study physical quantities in which external gauge field lines attach themselves onto matter particle contours we fully expect to encounter analogous computational features, e.g. pinch singularities, in these situations as well. We plan to report on such calculations in the near future.

6. First application: recovery of Feynman diagrammatic expansion to 2nd order

We have already pointed out that nothing is lost to the original content of the field theoretical system on the account of manipulations that have taken place for the purpose of casting it into its worldline form. As a first illustration — which, at the same time, will enable us to see the corresponding particle-based computational tools

[8] As an explicit illustration of this occurrence the reader is referred to the next section where a computation pertaining to $G^{(2)}(x,y)$ will be carried out.

at work — we intend, in this section, to prove the equivalence between the worldline and Feynman perturbative expansions for the fermionic propagator, to second order. Given the notable calculational advantages that have already been established for non-abelian gauge field theories in connection with closed particle contours [14, 7, 27], one is encouraged to think that our present extension will lead to efficient ways for perturbative estimates, to a given order, for processes involving open matter particle contours as well.

Let us commence our considerations by dispensing with the 0th order term of the fermionic propagator. As per our discussion in the previous section, we expect no contribution from the spin factor in the super particle action. Indeed, the only available four-vector in the free propagation case is $u_\mu = \frac{1}{T}(x-y)_\mu$ and there is no way to generate antisymmetric combinations from it. The relevant worldline expression reads

$$iG^{(0)}(x,y) = \int_0^\infty dT e^{-Tm^2} \int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau) \left[m - \frac{1}{2}\gamma \cdot \dot{x}(T)\right] e^{-\frac{1}{4}\int_o^T d\tau \dot{x}^2(\tau)} . \qquad (6.1)$$

The solution of the classical equations of motion is, simply, $x^{cl}(\tau) = \eta_\mu$, with η_μ as in eq. (5.9). Under the substitution $x(\tau) \to x^{cl}(\tau) + x(\tau)$ path integration is performed with the same action but with boundary conditions $x(0) = x(T) = 0$. Given that the classical value of the action is $\frac{1}{4}Tu^2$ and in view of (5.10) we finally obtain

$$\begin{aligned} iG^{(0)}(x,y) &= \int_0^\infty dT \frac{e^{-Tm^2}}{(4\pi T)^{D/2}} \left[m - \frac{1}{2}\gamma \cdot u\right] e^{-\frac{1}{4}Tu^2} \\ &= \int_0^\infty dT\, e^{-Tm^2} \int \frac{d^D q}{(2\pi)^D} e^{-iq(x-y)-Tq^2} (m - i\gamma \cdot q) . \end{aligned} \qquad (6.2)$$

Integration over dT finally produces the free fermionic propagator.

Focusing our attention to the mass containing term I_m in $iG^{(2)}(x,y)$ we first recall that spin factor contributions are already contained in (4.17). Moreover, we have carried out, in the previous section, the computation of relevant expectation values with respect to the bosonic action given by eq. (5.7). Putting everything, that has already been worked out, together we obtain

$$\begin{aligned} I_m = -mg^2 c_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \int_0^\infty \frac{dT}{(4\pi T)^{D/2}} e^{-Tm^2} \int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2 - \tau_1) \times \\ \times \left\{ D\ddot{G}_B(\tau_1,\tau_2) + [u + ik\dot{G}_B(\tau_1,\tau_2)]^2 - (D-1)k^2 G_F^2(\tau_1,\tau_2) \right\} \times \\ \times \left[-ik \cdot u(\tau_2 - \tau_1) - k^2 G_B(\tau_1,\tau_2) - \frac{T}{4}u^2 \right] . \end{aligned} \qquad (6.3)$$

Next, re replace $\ddot{G}_B$ an integration by parts:

$$\int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2-\tau_1)\ddot{G}_B(\tau_1,\tau_2)\exp\left[-ik\cdot u(\tau_2-\tau_1)-k^2G_B(\tau_1,\tau_2)\right] = \tag{6.4}$$
$$= \int_0^T d\tau_2 \dot{G}_B(\tau_2,0)\exp\left[-ik\cdot u\tau_2 - k^2 G_B(\tau_2,0)\right] + \int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2-\tau_1)\times$$
$$\times\left[k^2\dot{G}_B^2(\tau_1,\tau_2)-ik\cdot u\dot{G}_B(\tau_1,\tau_2)\right]\exp\left[-ik\cdot u(\tau_2-\tau_1)-k^2G_B(\tau_1,\tau_2)\right].$$

For the reader who is acquainted with closed-contour worldline manipulations the analogy with the above practice is familiar; simply the exponential has more terms for open lines. The important point is that in both situations the integration by parts is facilitated by the adopted regularization, namely $\dot{G}_B(\tau,\tau)=0$.

Making the variable change $\tau_2-\tau_1=\alpha T$ and $\tau_2+\tau_1=2\beta T$ we readily determine

$$I_m = -mg^2c_F\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2}\int_0^\infty\frac{dT}{(4\pi T)^{D/2}}T^2e^{-Tm^2}\int_0^1 d\alpha\,\times$$
$$\times\left\{(1-\alpha)\left[-4(D-1)k^2\alpha(1-\alpha)-(D-2)ik\cdot u(1-2\alpha)+u^2+\frac{2D}{T}\right]-\frac{D}{T}\right\}\times$$
$$\times\exp\left[-ik\cdot uT\alpha-k^2T\alpha(1-\alpha)-\frac{T}{4}u^2\right]. \tag{6.5}$$

Some comments are in order with respect to what has taken place in producing the above result. The most obvious one is that α assumes the role of a Feynman parameter as per the standard diagrammatic manipulation. Given its introduction the following practical rules of substitution, with regard to the original worldline expression, emerge

$$\dot{G}_B(\tau_1,\tau_2)=1-2\alpha\,,\qquad \dot{G}_B^2(\tau_1,\tau_2)-\dot{G}_F^2(\tau_1,\tau_2)=-4\alpha(1-\alpha)\,. \tag{6.6}$$

The last relation explicitly underlines the absence of divergencies in the integral over the Feynman parameter.

Turning our attention to the γ-matrix part, I_γ, of the second order propagator we employ the following relations pertaining to expectation values

$$\langle\gamma\cdot\dot{x}(T)\rangle = \frac{1}{(4\pi T)^{D/2}}\gamma\cdot\dot{x}^{cl}(T)\,,$$
$$\langle k\cdot\dot{x}(T)\gamma\cdot\dot{x}(\tau_i)\rangle = \frac{1}{(4\pi T)^{D/2}}\left[\gamma\cdot k\ddot{G}_B(\tau_i,T)+k\cdot\dot{x}^{cl}(T)\gamma\cdot\dot{x}^{cl}(\tau_i)\right],\qquad i=1,2\,,$$
$$\langle\dot{x}(T)\cdot\dot{x}(\tau_i)\rangle = \frac{1}{(4\pi T)^{D/2}}\left[D\ddot{G}_B(\tau_i,T)+\dot{x}^{cl}(T)\cdot\dot{x}^{cl}(\tau_i)\right],\qquad i=1,2 \tag{6.7}$$

and

$$\langle\gamma\cdot\dot{x}(T)\dot{x}(\tau_1)\cdot\dot{x}(\tau_2)\rangle = \frac{1}{(4\pi T)^{D/2}}\left[D\gamma\cdot\dot{x}^{cl}(T)\ddot{G}_B(\tau_1,\tau_2)+\gamma\cdot\dot{x}^{cl}(\tau_1)\ddot{G}_B(\tau_2,T)+\right.$$
$$\left.+\gamma\cdot\dot{x}^{cl}(\tau_2)\ddot{G}_B(\tau_1,T)+\gamma\cdot\dot{x}^{cl}(T)\dot{x}^{cl}(\tau_1))\cdot\dot{x}^{cl}(\tau_2)\right].$$

As before, we integrate by parts and make the variable change which introduces the Feynman parameter α. Doing so we are led to the following result, in momentum space,

$$\tilde{I}_\gamma(q) = ig^2 c_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \int_0^\infty dT T^2 \int_0^1 d\alpha (1-\alpha) \exp\left[-T(m^2+q^2-2q\cdot k+\alpha k^2)\right] \times$$
$$\times \left\{\gamma\cdot q\left[(D+2)m^2+(D-2)q^2-2(D-2)q\cdot k\right]+\gamma\cdot k(D-2)(m^2+q^2)\right\}. \tag{6.8}$$

Combining (6.3) and (6.7) we finally obtain

$$\begin{aligned}
\tilde{G}^{(2)} &= -g^2 c_F \frac{1}{(q^2+m^2)^2} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \frac{1}{(q-k)^2+m^2} \times \\
&\quad \times \Big\{ m\left[Dm^2+(D-4)q^2-2(D-2)q\cdot k\right] - \\
&\quad - i\gamma\cdot q\left[(D+2)m^2+(D-2)q^2-2(D-2)q\cdot k\right] - i\gamma\cdot k(D-2)(m^2+q^2)\Big\} \\
&= \frac{1}{m-i\gamma\cdot q}\left[-g^2 c_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \gamma_\mu \frac{m-i(\gamma\cdot q-\gamma\cdot k)}{(q-k)^2+m^2}\gamma_\mu\right] \frac{1}{m-i\gamma\cdot q},
\end{aligned} \tag{6.9}$$

which explicitly shows that the worldline perturbative, to second order, term for the fermionic propagator coincides with the one obtained via the Feynman diagrammatic expansion.

7. One loop calculation in the fermionic propagator

In this section we shall apply the worldline approach to a loop calculation pertaining to an open line which extends a recent closed line result obtained in [28], for a good review see ref. [29]. Specifically, by going back to (4.4) we shall perform the loop integration, equivalently, the integration over $d^D k$, thereby extracting an expression for the second order contribution to the fermionic propagator in parametric form. So as not to bore the reader with endless manipulations we shall restrict our attention to the mass term entering (4.4) which reads

$$I_m = -g^2 c_F \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \int_0^\infty dT e^{-Tm^2} \int_0^T d\tau_1 \int_0^T d\tau_2 \theta(\tau_2-\tau_1) \int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau) \times \tag{7.1}$$
$$\times\left\{\dot{x}(\tau_2)\cdot\dot{x}(\tau_1) - G_F^2(\tau_1,\tau_2)(D-1)k^2\right\} \exp\left[-\frac{1}{4}\int_0^T d\tau \dot{x}^2 - ik\cdot(x(\tau_1)-x(\tau_2))\right].$$

Introducing a small mass λ for infrared protection, as well as a mass μ for the purposes of dimensional regularization we perform the k-integrations making use of the formulae

$$\mu^{4-D}\int \frac{d^D k}{(2\pi)^D} \frac{(k^2)}{k^2+\lambda^2} e^{-ik\cdot x} = \frac{\mu^{4-D}}{(4\pi)^{D/2}} \int_0^\infty dL\, L^{-D/2} e^{-\lambda^2 L} \left(\frac{D}{2L} - \frac{x^2}{4L^2}\right) e^{-x^2/4L}, \tag{7.2}$$

where the parentheses around k^2 serve to point out that for numerator equal to 1 the terms in parentheses inside the integral on the right hand side are also substituted by 1.

Isolating the path integral component of I_m, i.e.

$$\mathcal{I} = \int_{\substack{x(0)=x\\ x(T)=y}} \mathcal{D}x(\tau) \left[\dot{x}(\tau_2)\cdot\dot{x}(\tau_1) + \frac{D-1}{4L^2} G_F^2(\tau_1,\tau_2)(x(\tau_2)-x(\tau_1))^2 - \right.$$
$$\left. - \frac{(D-1)D}{2L} G_F^2(\tau_1,\tau_2)\right] \times$$
$$\times \exp\left[-\frac{1}{4}\int_0^T d\tau\, \dot{x}^2 - \frac{1}{4L}(x(\tau_1)-x(\tau_2))^2\right], \tag{7.3}$$

we find ourselves having to calculate expectation values of bosonic super-particle field coordinates with respect to the action

$$S = \frac{1}{4}\int_0^T d\tau \dot{x}^2 + \frac{1}{L}(x(\tau_2)-x(\tau_1))^2 \,. \tag{7.4}$$

Leaving, for what follows, a number of technical details to the appendix we present the equations of motion in the form

$$\ddot{x}^{cl}_\mu(\sigma) = \frac{1}{L}\int_0^T d\sigma' B(\sigma',\sigma) x^{cl}_\mu(\sigma')\,, \tag{7.5}$$

where

$$B(\sigma_1,\sigma_2) = [\delta(\sigma_1-\tau_1) - \delta(\sigma_1-\tau_2)][\delta(\sigma_2-\tau_1) - \delta(\sigma_2-\tau_2]\,. \tag{7.6}$$

Next, we introduce the Green's function $\Delta^{(1)}(\sigma_1,\sigma_2)$ by

$$\int_0^T d\sigma \left[\delta(\sigma_1-\sigma)\frac{\partial^2}{\partial\sigma^2} - \frac{1}{L}B(\sigma,\sigma_1)\right]\Delta^{(1)}(\sigma,\sigma_2) = -\delta(\sigma_1-\sigma_2)\,, \tag{7.7}$$

obeying the boundary conditions $\Delta^{(1)}(0,\sigma) = \Delta^{(1)}(T,\sigma) = 0$.

The solutions to the classical equations of motion are readily determined as

$$x^{cl}_\mu(\sigma) = \eta_\mu(\sigma) - \frac{1}{L}(\tau_2-\tau_1)u_\mu\left[\Delta^{(1)}(\sigma,\tau_2) - \Delta^{(1)}(\sigma,\tau_1)\right], \tag{7.8}$$

with $\eta_\mu(\sigma)$ as given in (5.9).

The corresponding expression for the classical action is

$$S^{cl} = \frac{T}{4}u\cdot\dot{x}^{cl}(T) = \frac{Tu^2}{4} + \frac{1}{4}u^2\frac{(\tau_2-\tau_1)^2}{L+G_B(\tau_1,\tau_2)}\,. \tag{7.9}$$

Making the substitution $x\to x+x^{cl}$ we have that

$$\begin{aligned}\langle\dot{x}(\tau_1)\dot{x}(\tau_2)\rangle &\to \langle\dot{x}(\tau_1)\dot{x}(\tau_2)\rangle_o + \dot{x}^{cl}(\tau_1)\dot{x}^{cl}(\tau_2)\,,\\ \langle(x(\tau_2)-x(\tau_1))^2\rangle &\to \langle(x(\tau_2)-x(\tau_1))^2\rangle_o + (x^{cl}(\tau_2)-x^{cl}(\tau_1))^2\,,\end{aligned} \tag{7.10}$$

where

$$\langle A\rangle_o \equiv M_o^{-1}\int_{x(0)=(x(T)=0}\mathcal{D}x(\tau)Ae^{-S[x]}\,, \tag{7.11}$$

with $S[x]$ as in (7.4) and M_o^-1 the usual renormalization factor.

From (7.8) we readily determine

$$\dot{x}^{cl}(\tau_1)=\dot{x}^{cl}(\tau_2)=u_\mu+\frac{1}{2}u_\mu(\tau_1-\tau_2)\frac{\dot{G}_B(\tau_1,\tau_2)}{L+G_B(\tau_1,\tau_2)} \tag{7.12}$$

and

$$\left[x^{cl}(\tau_2)-x^{cl}(\tau_1)\right]^2=(\tau_2-\tau_1)^2u^2\left(\frac{L}{L+G_B(\tau_1,\tau_2)}\right)^2. \tag{7.13}$$

In the appendix we establish that

$$\mathcal{N}^{-1}\langle\dot{x}(\tau_1)\dot{x}(\tau_2)\rangle_o=D\ddot{G}_B(\tau_1,\tau_2)-\frac{D}{2}\frac{\dot{G}_B^2(\tau_1,\tau_2)}{L+G_B(\tau_1,\tau_2)} \tag{7.14}$$

and

$$\mathcal{N}^{-1}\langle(x(\tau_2)-x(\tau_1))^2\rangle_o=2D\frac{LG_B(\tau_1,\tau_2)}{L+G_B(\tau_1,\tau_2)}, \tag{7.15}$$

where $\mathcal{N}$ is also defined in the appendix.

Putting everything together we obtain (we suppress the τ_1,τ_2 of the G's for notational economy)

$$\begin{aligned}I_m=&-mg^2\frac{c_F}{(4\pi)^D}\mu^{4-D}\int_0^\infty dLe^{-\lambda^2L}\int_0^\infty dTT^{-D/2}e^{-Tm^2}\int_0^T d\tau_1\int_0^T d\tau_2\frac{\theta(\tau_2-\tau_1)}{(L+G_B)^{D/2}}\times\\&\times\left\{D\ddot{G}_B-\frac{D}{2}\frac{\dot{G}_B^2}{L+G_B}+u^2\left[1+\frac{1}{2}(\tau_2-\tau_1)\frac{\dot{G}_B}{L+G_B}\right]^2+\right.\\&\left.+\frac{D-1}{4L^2}G_F^2\left[2D\frac{LG_B}{L+G_B}+(\tau_2-\tau_1)^2u^2\frac{L^2}{(L+G_B)^2}-\frac{(D-1)D}{2L}G_F^2\right]\right\}\times\\&\times\exp\left[-\frac{Tu^2}{4}-\frac{u^2}{4}\frac{(\tau_2-\tau_1)^2}{L+G_B}\right].\end{aligned} \tag{7.16}$$

After an integration by parts, which rids us of the presence of $\ddot{G}_B(\tau_1,\tau_2)$ in favor of the combination $\dot{G}_B^2-G_F^2$, we get, once performing the substitution $\tau_2-\tau_1=\alpha T$, $\tau_1+\tau_2=2\beta T$, $L\to LT$,

$$\begin{aligned}I_m=&-mg^2\frac{c_F}{(4\pi)^D}\mu^{4-D}\int_0^\infty dL\int_0^\infty dTT^{3-D}e^{-T(m^2+\lambda L)}\int_0^1 d\alpha\frac{1-\alpha}{[L+\alpha(1-\alpha)]^{D/2}}\times\\&\times\left[u^2(D-1)\frac{\alpha^3(1-\alpha)}{[L+\alpha(1-\alpha)]^2}-\frac{2D(D-1)}{T}\frac{\alpha(1-\alpha)}{L+\alpha(1-\alpha)}+\right.\\&\left.+\frac{(D-2)u^2}{2}\frac{\alpha(1-2\alpha)}{L+\alpha(1-\alpha)}+u^2+\frac{D}{T}\frac{1-2\alpha}{1-\alpha}\right]\times\\&\times\exp\left[-\frac{Tu^2}{4}-\frac{Tu^2}{4}\frac{\alpha^2}{L+\alpha(1-\alpha)}\right].\end{aligned} \tag{7.17}$$

The corresponding expression for the Fourier transform reads

$$\tilde{I}_m = -mg^2 \frac{c_F}{(4\pi)^D} \mu^{4-D} \int_0^\infty dL \int_0^\infty dT T^{3-D/2} \int_0^1 d\alpha \frac{1-\alpha}{(L+\alpha)^{D/2}} \times \tag{7.18}$$
$$\times \left[D(m^2 - q^2 + \lambda^2 L) - 2(D-2)Lq^2\right] \exp\left[-T\left(m^2 + \lambda^2 L - q^2 + \frac{\alpha^2}{L+\alpha} q^2\right)\right]$$
$$= -mg^2 \frac{c_F}{(4\pi)^D} \mu^{4-D} \int_0^1 d\alpha \frac{D(m^2 - \alpha^2) + 2(D-2)(1-\alpha)q^2}{[m^2\alpha + (1-\alpha)\lambda^2 + q^2\alpha(1-\alpha)]^{2-D/2}} \Gamma(2 - D/2)\,.$$

As already mentioned, we shall refrain from working out the gamma-matrix part of the propagator. The point is that our methods have produced a one loop result in a parametrized form pertaining to a configuration with an *open* fermionic line, thereby extending the scope and use of the approach and methodology of ref. [7].

8. Concluding remarks

Our efforts, in this work, have centered around the role of the spin factor in the worldline approach to gauge theories with fermionic matter fields. Placing specific emphasis on open particle path configurations we were able to accommodate the spin attribute of the propagating entity into a geometrical type of expression (spin factor) which registers as a well defined weight factor in the path integral. Further to the point, its presence facilitates the extraction of simple computational rules which, at the perturbative level at least, produce algorithms of the Bern-Kosower type [14] while extending, at the same time, their applicability to fermionic Green's functions. On the other hand, the dynamics of the particle-based casting of the system amounts to emission and absorption of gauge field modes by the matter particle which is solely controlled by a Wilson line operator. Our scheme thereby achieves a neat separation between geometrical and dynamical aspects of physical descriptions in gauge field theories. To summarize, our formulation has not only made it possible to accommodate open fermionic paths, equivalently, Green's functions, into worldline considerations but has produced computational rules that have clear as well as simple interpretations.

The fact that our approach has identified the Wilson line operator as the sole agent of the dynamics operating in the system is worth some further comments. This occurrence has proven to be of central importance to the dynamical considerations we have been pursuing in connection to long distance properties of QCD [10]–[12]. The wider realization that Wilson line operators carry the dynamics of eikonally-based considerations in QCD is, actually, well founded in a variety of approaches which aim at factorizing soft contributions to quantities of physical interest, such as form factors and parton-parton scattering amplitudes [30]–[35]. A marked contrast is that in the worldline approach to QCD Wilson lines enter directly as an indigenous component of the action whereas in standard field theoretical approaches they are brought in,

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within the context of an operator formalism, for the purpose of representing quarks as ordered exponentials of the gluon field connected at the point where the hard scattering occurs. In other words, they enter as sort of 'tails' attached to the quark field operator for the purpose of accounting for long distance behavior. Our present work further establishes the role of the Wilson line operators as the quantity which carries the dynamics in a gauge field theory even beyond soft effects.

Be that as it may, our immediate plans are to study situations wherein external gluonic fields attach themselves to an open fermionic line thereby extending the present considerations to vertex functions and 'Compton' amplitudes. We expect to report on progress along these lines soon.

A. Manipulations involving bosonic propagators

We display, below, some interim steps associated with the derivation of formulas given in section 7. Consider the particle-based action functional S as registered in eq. (7.4). Casting it in the form

$$S = \frac{1}{4}\int_0^T d\tau \dot{x}^2(\tau) + \frac{1}{4L}\int_0^T d\sigma_1 \int_0^T d\sigma_2 B(\sigma_1, \sigma_2|\tau_1, \tau_2) x(\sigma_1)\cdot x(\sigma_2)\,, \tag{A.1}$$

with $B(\sigma_1, \sigma_2|\tau_1, \tau_2)$ as given by eq. (7.6), we are led to the equations of motion given by eq. (7.5) in the text.

Considering a solution of the form $x_\mu = \eta_\mu + z_\mu$, with η_μ as in (5.9) we are led to the relation (we drop, as in the text, the τ-arguments in B)

$$\int_0^T d\sigma' \left[\delta(\sigma-\sigma')\frac{\partial^2}{\partial\sigma'^2} - \frac{1}{L}B(\sigma', \sigma)\right] z_\mu(\sigma') = [\delta(\sigma-\tau_2)-\delta(\sigma-\tau_1)](\tau_1-\tau_2)\frac{u_\mu}{L}\,, \tag{A.2}$$

which justifies the introduction of $\Delta^{(1)}(\sigma, \sigma')$ according to (7.7). One, now, readily determines

$$x_\mu^{cl}(\sigma) = \eta_\mu - \int_0^T d\sigma' \Delta^{(1)}(\sigma, \sigma')[\delta(\sigma'-\tau_2) - \delta(\sigma'-\tau_1)](\tau_1-\tau_2)\frac{u_\mu}{L}\,, \tag{A.3}$$

which produces eq. (7.8) in the text.

Employing the notation $\left[\delta(\sigma-\sigma')\frac{\partial^2}{\partial\sigma'^2} - \frac{1}{L}B(\sigma', \sigma)\right] \equiv \langle\sigma|(\partial^2 - \frac{B}{L})|\sigma'\rangle$ we find, following the procedure applied in ref. [36] for the closed contour case,

$$\begin{aligned}-\Delta^{(1)}(\sigma, \sigma') &= \langle\sigma|\left(\partial^2 - \frac{B}{L}\right)^{-1}|\sigma'\rangle = \langle\sigma|\left(1 - \frac{B}{L}\right)^{-1}\partial^{-2}|\sigma'\rangle \\ &= \langle\sigma|\partial^{-2}|\sigma'\rangle + \frac{1}{L}\langle\sigma|\partial^{-2}B\partial^{-2}|\sigma'\rangle + \frac{1}{L^2}\langle\sigma|\partial^{-2}B\partial^{-2}B\partial^{-2}|\sigma'\rangle + \cdots\end{aligned} \tag{A.4}$$

with the first term in the series to be denoted, for the present purposes, by $\Delta^{(0)}(\sigma, \sigma')$; it actually coincides with the free bosonic propagator, cf. eq. (5.1).

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Straight forward manipulations on the matrix elements entering the above series result to the following, closed-form, relation

$$\Delta^{(1)}(\sigma,\sigma')=\Delta^{(0)}(\sigma,\sigma')-\frac{[\Delta^{(0)}(\sigma,\tau_1)-\Delta^{(0)}(\sigma,\tau_2)][\Delta^{(0)}(\tau_1,\sigma')-\Delta^{(0)}(\tau_2,\sigma')]}{L+G_B(\tau_1,\tau_2)}\,. \tag{A.5}$$

Liberal use of (A.2) and (A.4) leads to the relation

$$x^{cl}_\mu(\tau_2)-x^{cl}_\mu(\tau_1)=(\tau_2-\tau_1)\frac{L}{L+G_B(\tau_1,\tau_2)}\,, \tag{A.6}$$

which, in turn, leads to eq. (7.13) in the text.

From (A.2) we obtain, after some straight forward manipulations, for the derivative of the classical bosonic particle fields

$$\dot{x}^{cl}_\mu(\sigma)=u_\mu-\frac{1}{2}u_\mu(\tau_1-\tau_2)\frac{\dot{G}_B(\sigma,\tau_1)-\dot{G}_B(\sigma,\tau_2)}{L+G_B(\tau_1,\tau_2)}\,, \tag{A.7}$$

which justifies eq. (7.12).

Turning our attention to expectation values, let us first furnish an outline derivation of (7.14). Under the substitution $x\to x^{cl}+x$ we have that

$$\langle\dot{x}(\tau_1)\cdot\dot{x}(\tau_2)\rangle=\left[\langle\dot{x}(\tau_1)\cdot\dot{x}(\tau_2)\rangle_o+\dot{x}^{cl}(\tau_1)\cdot\dot{x}^{cl}(\tau_2)\langle 1\rangle_o\right]e^{-S^{cl}}\,, \tag{A.8}$$

where

$$\langle 1\rangle_o=\int_{x(0)=x(T)=0}\mathcal{D}x(\tau)\exp\left[\frac{1}{4}\int_0^T d\tau\dot{x}^2-\frac{1}{4L}(x(\tau_2)-x(\tau_1))^2\right]. \tag{A.9}$$

One easily obtains

$$\langle 1\rangle_o=\frac{1}{(4\pi T)^{D/2}}\left[\frac{L}{L+G_B(\tau_1,\tau_2)}\right]^{D/2}. \tag{A.10}$$

On the other hand

$$\begin{aligned}\langle\dot{x}(\tau_1)\cdot\dot{x}(\tau_2)\rangle_o=&-\frac{\delta^2}{\delta J_\mu(\tau_1)\delta J_\mu(\tau_2)}\int_{x(0)=x(T)=0}\mathcal{D}x(\tau)\times\\&\times\exp\left[\frac{1}{4}\int_0^T d\tau\dot{x}^2-\frac{1}{4L}\int_0^T d\sigma_1\int_0^T d\sigma_2 B(\sigma_1,\sigma_2)-\right.\\&\left.\left.-i\int_0^T d\tau J(\tau)\cdot\dot{x}(\tau)\right]\right|_{J=0}.\end{aligned} \tag{A.11}$$

Now, from (A.4) we determine

$$\frac{\partial^2}{\partial\sigma\partial\sigma'}\Delta^{(1)}(\sigma,\sigma')=\frac{1}{2}\ddot{G}_B(\sigma,\sigma')-\frac{1}{4}\frac{\left[\dot{G}_B(\sigma,\tau_2)-\dot{G}_B(\sigma,\tau_1)\right]\left[\dot{G}_B(\sigma',\tau_2)-\dot{G}_B(\sigma',\tau_1)\right]}{L+G_B(\tau_1,\tau_2)}. \tag{A.12}$$

which facilitates the carrying out of the (functional) gaussian integration in (A.11). One finds

$$\langle \dot{x}(\tau_1)\cdot\dot{x}(\tau_2)\rangle_o = \frac{1}{(4\pi T)^{D/2}}\left(\frac{L}{L+G_B(\tau_1,\tau_2}\right)^{D/2} D\left[\ddot{G}_B(\tau_1,\tau_2)-\frac{1}{2}\frac{\dot{G}_B^2(\tau_1,\tau_2)}{L+G_B(\tau_1,\tau_2)}\right] \tag{A.13}$$

as per eq. (7.14) in the text.

Finally, eq. (7.15) is deduced as follows

$$\begin{aligned}\langle (x(\tau_2)-x(\tau_1)^2\rangle_o &= \langle (x^2(\tau_2)\rangle_o + \langle (x^2(\tau_1)\rangle_o - 2\langle (x(\tau_2)\cdot(x(\tau_1)\rangle_o \\ &= 2D\left[\Delta^{(1)}(\tau_2,\tau_2)+\Delta^{(1)}(\tau_1,\tau_1)-2\Delta^{(1)}(\tau_1,\tau_2)\right]\langle 1\rangle_o \\ &= 2D\frac{LG_B(\tau_1,\tau_2)}{L+G_B(\tau_1,\tau_2)}\langle 1\rangle_o\,. \end{aligned} \tag{A.14}$$

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